

Evidence of double magicity of $N = Z$ nuclei near the rp-process path

M.M. Sharma*

Physics Department, Kuwait University, Kuwait 13060, Kuwait

J.K. Sharma

Physics Department, St. John's College, Agra-282002, India

(Dated: July 6, 2009)

Abstract

$N = Z$ nuclei above Ni are understood to be waiting-point nuclei in the rp-process nucleosynthesis. Investigating the experimental isotope shifts in Kr isotopes near the proton drip-line, we have discovered that $N = Z$ rp-process nuclei ^{68}Se , ^{72}Kr , ^{76}Sr and ^{80}Zr exhibit a significant shell gap both at the proton and neutron numbers in the deformed space with the consequence that pairing correlations for protons and neutrons vanish, thus lending a double-magic character to these nuclei. A significant number of nuclei in this region are also shown to exhibit neutron magicity at $N = 34, 36, 38$, and 40 in the deformed space.

PACS numbers: 21.10.Ft, 21.60.Cs, 21.60.Jz, 26.30.-k, 26.30.Hj, 27.50.+e

In rapid-proton-capture (rp-process) in x-ray bursters, $N = Z$ nuclei above Ni ($Z = 28$) are understood to play an important role in producing a part of x-ray flux [1, 2, 3]. A number of these nuclei are considered to be waiting-point due to inability of these nuclei to capture a proton, thus hindering the rp-process. Consequently, peaks in abundances of the $N = Z$ nuclei ^{68}Se , ^{72}Kr , ^{76}Sr and ^{80}Zr among others have been shown to arise in astrophysical rp-process in x-ray bursters [2].

Precise measurements [4, 5, 6, 7] of nuclear masses have established the waiting-point character of the $N = Z$ nuclei ^{68}Se , ^{72}Kr , ^{76}Sr , and ^{80}Zr . On ^{64}Ge , however, there is no consensus as yet whether it is a waiting-point nucleus [8]. Negative values of Q_p for a proton capture are cited as evidence for the waiting-point character of nuclei. This is interesting, since no known magic number is present in this region in contrast to waiting-point nuclei arising due to the major magic numbers along the r-process path. This poses a theoretical challenge to understand the structure of the waiting-point nature of rp-process nuclei responsible for x-ray bursts.

The structure of these nuclei is far from certain. What is known is that nuclei in this region are highly deformed [9, 10, 11] until the major magic number approaches at $Z = N = 50$ where the rp-process is predicted to terminate [2]. In this letter, we examine the experimental isotope shifts in proton-rich Kr nuclei near the rp-process path within the framework of the relativistic Hartree-Bogoliubov theory in deformed space. Consequently, we report on the double-magic structure that we have discerned for $N = Z$ nuclei near the rp-process path.

Isotope shifts in nuclei reveal shell effects across a major shell gap [12], for instance, for the Pb isotopes in terms of a kink about the magic number [13]. For other isotopic chains such as Kr and Sr, such a behavior is shrouded by deformation of nuclei. In this work, we investigate the isotope shifts Δr_c^2 in Kr nuclei. The experimental values (with $N = 50$ as a reference point) for the chain are known for isotopes down to $N = 36$ with a significant precision from laser spectroscopy measurements [14] (see Fig 1). The nuclide ^{72}Kr ($N = 36$) is very close to the proton drip line. The salient feature of these data (Fig. 1) is the monotonous increase of Δr_c^2 from $N = 50$ down to $N = 40$ due to an increasing deformation in going to the lighter isotopes. A similar feature is seen for isotopes heavier than $N = 50$. However, the most interesting feature that is demonstrated by the experimental data is that Δr_c^2 for ^{74}Kr is nearly zero and that it becomes strongly negative for ^{72}Kr . The magnitude of the isotope shift or in other words, the charge radius of a nucleus with respect to that of

the reference nucleus is mainly reflective of the deformation of a nucleus. To the first order, it can be approximated as

$$\Delta r_c^2 = \langle r_c^2 \rangle_s \frac{5}{4\pi} \delta \beta_2^2, \quad (1)$$

where $\langle r_c^2 \rangle_s$ is the mean-square charge radius of the spherical nucleus and β_2 is the quadrupole deformation of the deformed one. This excludes any effects due to shell structure. The downward trend of the experimental isotope shift below $N = 40$ whilst retaining β_2 in the same range would imply a structural factor. Here, we investigate this factor that leads to a decreasing or even a negative isotope shift.

We examine the isotope shifts of Kr nuclei within the framework of the relativistic mean-field theory. In this work, we employ the standard RMF Lagrangian with the exchange of σ , ω and ρ mesons between the nucleons. The corresponding Lagrangian density which describes the nucleons as Dirac spinors moving in meson fields is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left(\not{p} - g_\omega \not{\omega} - g_\rho \not{\vec{\rho}} \vec{\tau} - \frac{1}{2} e (1 - \tau_3) \not{A} - g_\sigma \sigma - M_N \right) \psi \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ & + \frac{1}{2} g_4 (\omega_\mu \omega^\mu)^2 - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \end{aligned} \quad (2)$$

$U(\sigma)$ represents the conventional nonlinear σ potential. This Lagrangian includes the vector self-coupling of ω -meson represented by the coupling constant g_4 . In this work, we employ the Lagrangian set NL-SV1 with the vector self-coupling of ω meson. The force NL-SV1 was developed with a view to soften the high-density equation of state of nuclear matter and has been shown to improve the shell effects of nuclei along the stability line [15].

Deformed RMF+BCS calculations have been performed using an expansion of fermionic and bosonic wavefunctions in 20 oscillator shells. The pairing is included using the constant gap approximation with pairing gap being taken from the widely used prescription of $\Delta_{n(p)} = 4.8N^{-1/3}(Z^{-1/3})$ for open-shell nuclei [16]. The results of RMF+BCS calculations of charge radii obtained with NL-SV1 are shown in Fig. 1. The isotope shifts thus obtained show an increasing trend in going below $N = 50$ except for $N = 44$ where the experimental data is underestimated by the theory due to a relatively smaller deformation $\beta_2 \sim 0.10$ as compared to its neighbors. The experimental values are reproduced satisfactorily down to $N = 40$ as well as above $N = 50$. However, for the isotopes ^{74}Kr ($N = 38$) and ^{72}Kr ($N = 36$), there is a strong divergence of the theoretical values from the experimental data, though β_2 of

these nuclei is significantly larger than that for ^{76}Kr ($N = 40$). The isotope shift for ^{70}Kr ($N = 34$) shows a further increase. Experimental datum does not exist for this isotope.

Theoretically, the rapid increase in the charge radius of ^{74}Kr , ^{72}Kr and ^{70}Kr appears naturally as a consequence of the vicinity to the proton drip line. The BCS smearing of occupation probabilities across the Fermi surface which adjoins the continuum leads to a swelling of the charge radius. Evidently, a BCS description of nuclei close to the proton drip line runs contrary to the experimental data. The isotope shifts of Kr and Sr isotopes were investigated in a previous work [17] within the RMF+BCS formalism. Calculations performed with 12 oscillator shells gave rise to an apparently good agreement with the data [17]. This agreement is, however, fortuitous, for 12 shells do not suffice to encompass the configuration space and hence underestimate the charge radii of neutron-deficient nuclei significantly.

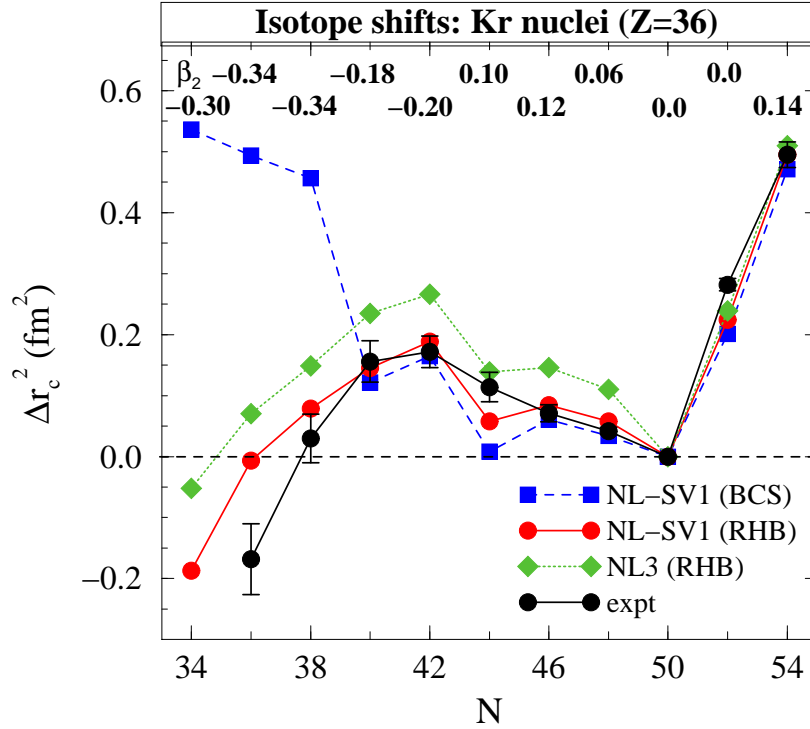


FIG. 1: The isotope shifts Δr_c^2 of Kr chain obtained from deformed RMF+BCS and RHB calculations with NL-SV1. Deformation β_2 for the ground-state of nuclei obtained with NL-SV1 (RHB) are shown at the top of the figure. The experimental data [14] for Δr_c^2 are shown for comparison. The results with NL3 are also shown.

Notwithstanding the stark disagreement of the RMF+BCS calculations with the experimental isotope shifts, we have undertaken an inclusion of the pairing mechanism based upon Bogoliubov quasi-particle scheme in a deformed basis with a view to take into account the effects due to coupling of the Fermi surface to the continuum. The eigenstates are then eigenvectors of the generalized single-particle Hamiltonian containing the self-consistent mean-field and a pairing field $\hat{\Delta}$ representing the particle-particle correlations. Staying within the Hartree approximation for the mean-field, the relativistic Hartree-Bogoliubov equations are written as

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + m + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k, \quad (3)$$

where \hat{h}_D is the single-particle Dirac Hamiltonian with quasi-particle energies E_k and the chemical potential λ . Herein, the pairing field

$$\hat{\Delta}_{ab}(\mathbf{r}, \mathbf{r}') = \frac{1}{2} \sum_{cd} V_{abcd}(\mathbf{r}, \mathbf{r}') \kappa_{cd}(\mathbf{r}, \mathbf{r}') \quad (4)$$

is the sum over matrix elements $V_{abcd}(\mathbf{r}, \mathbf{r}')$ of a two-body pairing interaction and the corresponding pairing tensor is defined by

$$\kappa_{cd}(\mathbf{r}, \mathbf{r}') = \sum_{E_k > 0} U_{ck}^*(\mathbf{r}) V_{dk}(\mathbf{r}'). \quad (5)$$

The RHB equations are solved self-consistently in order to obtain eigensolutions and eigenvalues in a single quasi-particle basis. This is transformed to the canonical basis to obtain the desired observables. We have used the Gogny force D1S [18] for the pairing channel as in other works [19]. We have adjusted the strength of the pairing force in order to get a good agreement of the calculated binding energies with the experimental values for the chain of Sn isotopes. The superfluid Sn nuclei provide a best testing ground for calibration of the pairing strength. Consequently, the force NL-SV1 describes the ground-state binding energies over a large range of Sn isotopes very well.

Deformed RHB calculations with the pairing channel thus calibrated have been performed for the Kr isotopes using an expansion of Dirac spinors and mesons fields into 20 shells of an axially deformed oscillator potential. In each case, minimizations have been sought in the oblate and prolate regions of deformation. It is noticed that there is a shape-coexistence between an oblate and a prolate shape for several isotopes. The RHB calculations with

NL-SV1 provide a good description of the ground-state binding energies of Kr isotopes with a few divergences within 0.20%.

The results obtained on the isotope shifts with NL-SV1 with the deformed RHB are shown in Fig. 1. The RHB results from $N = 48$ towards $N = 40$ show an improvement over the BCS values including that for $N = 44$. The most interesting outcome of these calculations is the downward trend that arises for the isotopes below $N = 40$, a picture that contrasts sharply with the BCS pairing. NL-SV1 overestimates the isotope shift (charge radius) of ^{72}Kr ($N = 36$) slightly. This is due perhaps to a slightly higher deformation ($\beta_2 = -0.34$) obtained theoretically than is the case. There is no datum on ^{70}Kr . We predict a negative isotope shift for this nucleus. Results obtained with the set NL3 also reproduce the downward trend. The NL3 results, however, overestimate the experimental data.

The case of ^{72}Kr is noteworthy. It is a $N = Z$ nucleus that participates significantly in the rp-process. The Nilsson splitting of j -levels especially near the Fermi surface carves out a major shell gap both at $N = 36$ and $Z = 36$. Our results with the deformed RHB calculations show that pairing correlations vanish completely for protons and neutrons alike. This renders the nucleus ^{72}Kr as doubly magic in the deformed space. Thus, in spite of being close to the proton drip line, the absence of smearing engenders the decrease in the charge radius in stark contrast to the BCS result (see Fig. 1).

The isotopes ^{70}Kr and ^{74}Kr emerge with a major shell gap with vanishing pairing at the respective neutron numbers $N = 34$ and $N = 36$ in the ground state, thus exhibiting a neutron magicity only. This is, to a large extent, responsible for suppressing the charge radius of these isotopes.

With the advent of the double magicity in ^{72}Kr , we have also explored other $N = Z$ rp-process nuclei. Results of the deformed RHB calculations with NL-SV1 show that not only ^{72}Kr , but also the other $N = Z$ nuclei such as ^{68}Se , ^{76}Sr and ^{80}Zr exhibit larger shell gaps both at the respective neutron and proton numbers. In these nuclei pairing correlations vanish completely for neutrons and protons, which characterizes these nuclei also as doubly magic. Thus, the nuclei ^{68}Se , ^{72}Kr , ^{76}Sr and ^{80}Zr with $N = Z = 34, 36, 38$, and 40 , respectively, are endowed with a double magicity in the deformed space. It may be remarked that though deformed shell gaps at some of these N and Z numbers have been indicated in the literature (e.g. ref. [20]), these shell gaps have never been shown to qualify for a magicity.

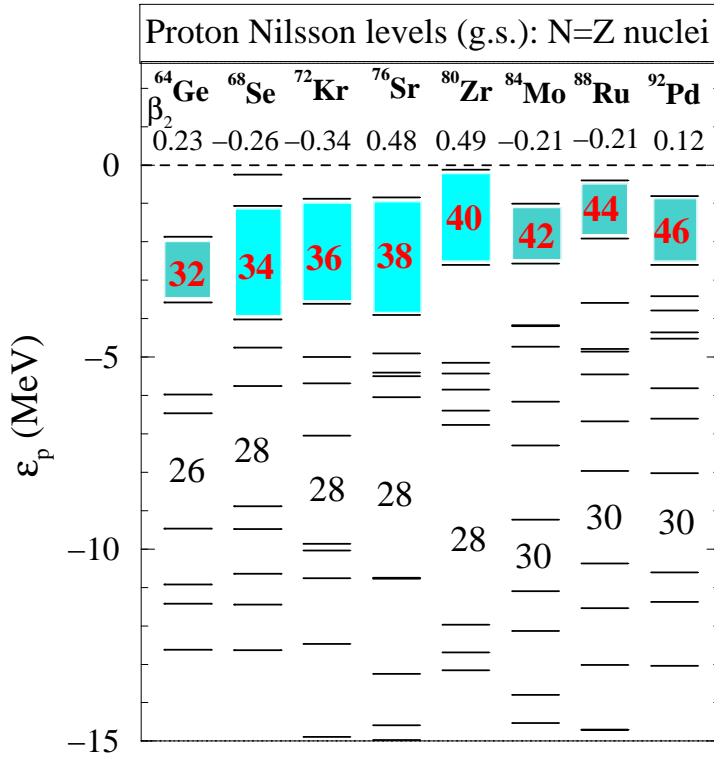


FIG. 2: Proton single-particle levels for the ground-state of $N = Z$ nuclei obtained with deformed RHB approach using the force NL-SV1. Nuclei with the vanishing pairing correlations (proton magicity) are highlighted with a larger shell gap in columns 2-5.

The ensuing single-particle structure of $N = Z$ nuclei is shown in Figs. 2 and 3, where the proton and neutron Nilsson single-particle levels obtained with the deformed RHB approach using NL-SV1 are displayed. The larger shell gaps at N and $Z = 34, 36, 38$, and 40 can be seen conspicuously for the aforesaid $N = Z$ nuclei. The corresponding β_2 for the ground-state is indicated in the upper part of the figures. It is noteworthy that ^{76}Sr and ^{80}Zr exhibit a large prolate deformation in the ground state. This is consistent with the experimental values deduced [21, 22].

The case of ^{68}Se deserves a mention. This nucleus has been shown experimentally to be a waiting-point nucleus beyond doubt [7]. Our results show that this nucleus has a second minimum with a prolate shape within 0.5 MeV of the oblate ground-state, thus exhibiting a shape-coexistence. Interestingly, the single-particle structure of the prolate state also exhibits a doubly magic character. In comparison, ^{64}Ge ($N = Z = 32$) shows shell gaps in neutrons and protons which can not be construed as a magic number. The corresponding

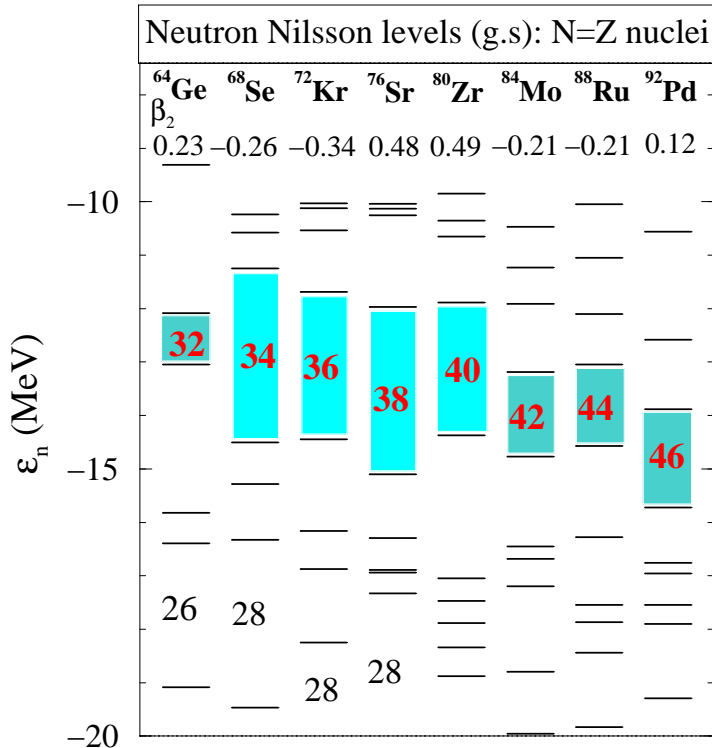


FIG. 3: Neutron single-particle levels for the ground-state of $N = Z$ nuclei obtained with deformed RHB approach using the force NL-SV1. Nuclei with the vanishing pairing correlations (neutron magicity) are highlighted in columns 2-5.

pairing energy for neutrons and protons is non-zero albeit significantly reduced as compared to a normal superfluid nucleus. Other $N = Z$ nuclei such as ^{84}Mo , ^{88}Ru , and ^{92}Pd also show a well-deformed shape in the ground state and a shell gap both in protons and neutrons near the Fermi surface similar in magnitude to that in ^{64}Ge with non-zero and yet significantly reduced pairing correlations. These nuclei are also in contention for being waiting-point nuclei experimentally [1]. However, our results do not lend a doubly-magic character to these nuclei.

We have examined the single-particle structure of other nuclei in the vicinity of the rp-process path from $N = Z = 32$ -40. Deformed RHB calculations have been performed for sets of nuclei as shown in Fig. 4. As is prevalent in this region, ground state of nuclei leads to a significant deformation for most of the nuclei explored. Some cases also show a shape-coexistence between an oblate and a prolate shape. Details of this study will be presented elsewhere. In Fig. 4 we show the nuclei which exhibit a neutron and/or a proton

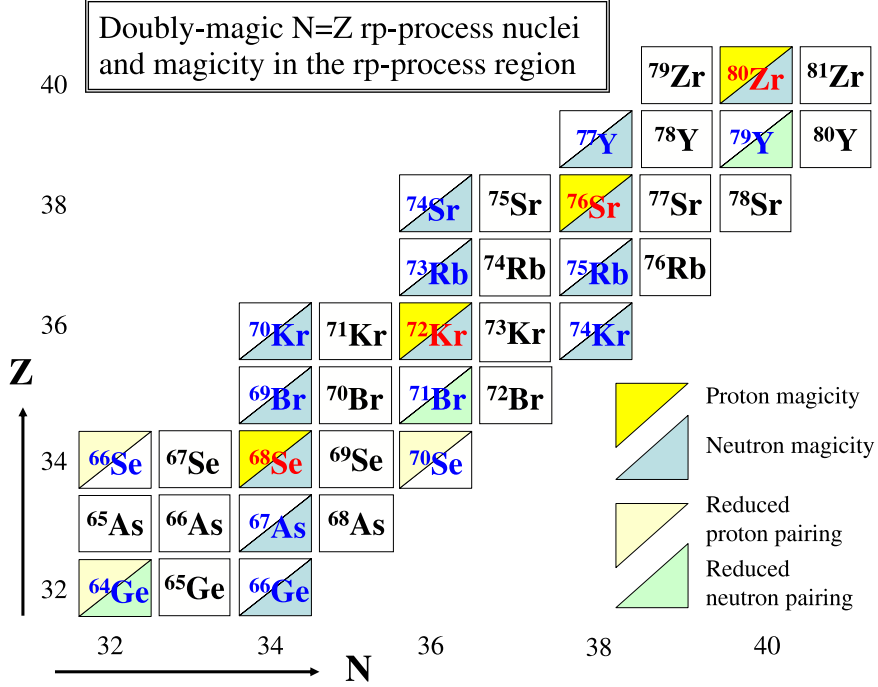


FIG. 4: The chart of $N = Z$ nuclei exhibiting double magicity along the rp-process path. Nuclides exhibiting neutron magicity can be seen in the columns with $N = 34, 36, 38$ and 40 . A few cases with the reduced pairing for neutrons and protons are also shown.

magicity. The four $N = Z$ nuclei ^{68}Se , ^{72}Kr , ^{76}Sr , and ^{80}Zr with double magicity stand out conspicuously.

We note that there is a preponderance of neutron magicity at several neutron numbers. This includes isotones with $N = 34, 36, 38$ and to a limited extent with $N = 40$. Surprisingly, a large number of nuclei in this region are amenable to a larger neutron shell gap at the Fermi surface in the Nilsson scheme with the vanishing pairing correlations. A few cases with significantly reduced neutron pairing correlations at $N = 32, 36$ and 40 with the pairing energy of ~ -1 to -2 MeV are also shown in Fig. 4. In contrast, there are no cases of proton magicity found other than those of four $N = Z$ doubly magic nuclei. Comparatively, the isotopes of ^{64}Ge , ^{66}Ge ($Z = 32$) and ^{70}Se ($Z = 34$) exhibit proton shell gaps with non-zero yet significantly reduced proton pairing energy. Thus, proton magicity in this region is rather subdued as compared to neutrons.

It is interesting to note that peaks in abundances of masses at $A = 64, 68, 72, 76$, and 80 were obtained in rp-process calculations [2] in an x-ray burst, testifying to an important role played by the $N = Z = 32-40$ nuclei. A double magicity of nuclei found in our work will

reinforce the waiting-point character to its namesake. It would be interesting to investigate as to how this attribute would influence the β -decay half-lives of nuclei and what would its effect on x-ray flux emanating from x-ray binaries be?

In conclusion, on the basis of our investigation of the experimental isotope shifts of Kr nuclei near the proton drip-line within the framework of the relativistic Hartree-Bogoliubov approach, we have found that $N = Z$ rp-process waiting-point nuclei ^{68}Se , ^{72}Kr , ^{76}Sr , and ^{80}Zr exhibit double magicity in the deformed space. It is also shown that there is a preponderance of nuclides exhibiting a neutron magicity at $N = 34, 36, 38$ and 40 in this region.

One of the authors (MMS) thanks Hendrik Schatz for useful discussions. This work is supported in part by Project SP04/04 of the Research Administration, Kuwait University.

* sharma@kuc01.kuniv.edu.kw

- [1] H. Schatz et al., Phys. Rep. **294**, 157 (1998).
- [2] H. Schatz et al., Phys. Rev Lett. **86**, 3471 (2001).
- [3] M. Wiescher and H. Schatz, Nucl Phys. A **693**, 269 (2001).
- [4] J.A. Clark et al., Phys. Rev Lett. **92**, 192501 (2004).
- [5] D. Rodriguez et al., Phys. Rev Lett. **93**, 161104 (2004).
- [6] J.A. Clark et al., Phys. Rev C **75**, 032801(R) (2007).
- [7] M.B. Gómez-Hornillos et al., Phys. Rev C **78**, 014311 (2008).
- [8] P. Schury et al., Phys. Rev C **75**, 055801 (2007).
- [9] A. Petrovici, K.W. Schmidt, and A. Faessler, Nucl Phys. A **605**, 290 (1996).
- [10] P. Sarriguren, R. Alvarez-Rodriguez, and E. Moya de Guerra, Eur. Phys. J. A **24**, 193 (2005).
- [11] K. Langanke, D. Dean, and W. Nazarewicz, Nucl Phys. A **728**, 109 (2003).
- [12] E.W. Otten, in *Treatise on Heavy-Ion Science*, Vol 7, D.A. Bromley (Ed.), 1989.
- [13] M.M. Sharma and G.A. Lalazissis, P. Ring, Phys. Lett B **317**, 9 (1993).
- [14] M. Keim et al., Nucl Phys. A **586**, 219 (1995).
- [15] M.M. Sharma, A.R. Farhan and S. Mythili, Phys. Rev C **61**, 054306 (2000).
- [16] P. Möller and J.R. Nix, Nucl Phys. A **536**, 221 (1992).
- [17] G.A. Lalazissis and M.M. Sharma, Nucl Phys. A **586**, 201 (1995).

- [18] J.F. Berger, M. Girod, and D. Gogny, Nucl Phys. A **428**, 32 (1984).
- [19] G.A. Lalazissis, D. Vretenar, and P. Ring, Nucl Phys. A **679**, 481 (2001).
- [20] W. Nazarewicz et al., Nucl Phys. A **435**, 397 (1985).
- [21] C.J. Lister et al., Phys. Rev Lett. **59**, 1270 (1987).
- [22] E. Nácher et al., Phys. Rev Lett. **92**, 232501 (2004).